Radiative Correction to the Transferred Polarization in Elastic Electron-Proton Scattering

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Abstract

Model independent radiative correction to the recoil proton polarization for the elastic electron–proton scattering is calculated within method of electron structure functions. The explicit expressions for the recoil proton polarization are represented as a contraction of the electron structure and the hard part of the polarization dependent contribution into cross–section. The calculation of the hard part with first order radiative correction is performed. The obtained representation includes the leading radiative corrections in all orders of perturbation theory and the main part of the second order next–to–leading ones. Numerical calculations illustrate our analytical results.

1 Introduction

It was proposed over 25 years ago [1] that recoil proton polarization in the elastic process $\vec{e} + p \rightarrow e + \vec{P}$, can be used to measure the proton electric form factor (G_{EP}). This method provides an alternative to the Rosenbluth separation and appears to be more sensitive to G_{EP} in the GeV–range of 4–momentum transfers (Q^2). Such measurements were done first at MIT-Bates [2] and later on extended to higher $Q^2 = 3.5$ GeV^2 at Jefferson Lab [3]. The latter experiment provided the first evidence of significant deviation of G_{EP} from the dipole form at higher Q^2.

In the recent Jefferson Lab experiment [3] the events corresponding to elastic process

$$\vec{e}^-(k_1) + P(p_1) \rightarrow e^-(k_2) + \vec{P}(p_2) \tag{1}$$

as well as radiative process

$$\vec{e}^-(k_1) + P(p_1) \rightarrow e^-(k_2) + \gamma(k) + \vec{P}(p_2) \tag{2}$$

have been analyzed.

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The main goal of these experiments is the measurement of the proton electric formfactor G_E. It can be done because the ratio of the longitudinal polarization of recoil proton to the transverse one in Born approximation is proportional to the ratio G_M/G_E [1] where G_M is the well known proton magnetic formfactor. This statement is valid if 3–vector of the longitudinal polarization has orientation along the recoil proton 3–momentum, and 3–vector of the transverse polarization is within the plane $(\vec{k}_1, \vec{p}_2)$. The interpretation of these high-precision experiments in terms of the proton electromagnetic formfactors G_M and G_E requires adequate theoretical calculations with a per cent accuracy or better. Such calculations must include the first order radiative corrections (RC) to the elastic cross–section (due to radiation of real soft and virtual photon) and full analysis of the radiative events. Moreover, leading higher order corrections have to be taken into account.

All the corresponding contributions can be joint within the framework of the electron structure function representation, which is a QED analog of the well known Drell–Yan representation [4]. This representation was applied before for the calculation of the RC to unpolarized electron–positron annihilation [5] and deep inelastic scattering [6] cross–sections.

In the present work we generalize the electron structure function representation for the case of scattering of polarized particles, namely for the analysis of the recoil proton polarization in elastic ep-scattering.

2   The leading approximation

The cross–section of the quasireal electron–proton scattering in the framework of the electron structure function method can be written as a contraction of two electron structure functions, that corresponds to the possibility to radiate hard collinear as well as virtual and soft photons and electron–positron pairs by both the initial and the scattered electron, and hard part of the cross–section that depends on shifted 4–momenta. This representation follows from the quasireal electron method [7] that is suitable for description of the collinear radiation.

In the problem considered here we will be interested in the spin dependent part of the cross–section only. For this case the corresponding representation can be written as

$$\frac{d\sigma^{\|,\perp}(k_1, k_2)}{d\,Q^2 d\,y} = \int\limits_{z_{1m}}^{1} d\,z_1 \int\limits_{z_{2m}}^{1} d\,z_2 D^{(p)}(z_1, L)\frac{1}{z_2^2}D^{(u)}(z_2, L)\frac{d\sigma^{\|,\perp(hard)}(\hat{k}_1, \hat{k}_2)}{d\,\hat{Q}^2 d\,\hat{y}} \ , \quad L = \ln\frac{Q^2}{m^2} \ , \quad (3)$$

where m is the electron mass,

$$\hat{k}_1 = z_1 k_1 \ , \ \hat{k}_2 = \frac{k_2}{z_2} \ , \ Q^2 = -(k_1 - k_2)^2 \ , \ \hat{Q}^2 = -(\hat{k}_1 - \hat{k}_2)^2 = \frac{z_1}{z_2}Q^2 \ , \quad (4)$$

$$y = \frac{2p_1(k_1 - k_2)}{V} \ , \ \hat{y} = 1 - \frac{1 - y}{z_1 z_2} \ , \ V = 2p_1 k_1 \ .$$

The electron structure function $D^{(p)}(z_1, L)$ is responsible for radiation by the initial polarized electron, whereas the function $D^{(u)}(z_2, L)$ describes radiation by the scattered unpolarized electron. The photonic contribution into the electron structure function is the same for polarized and unpolarized cases, but the contribution due to pair production differs in the singlet channel [8]. Therefore we can write

$$D^{(u)}(z, L) = D^\gamma(z, L) + D_N^{e^+e^-} + D_S^{e^+e^-(u)} \ , \quad (5)$$

$$D^{(p)}(z, L) = D^\gamma(z, L) + D_N^{e^+e^-} + D_S^{e^+e^-\,(p)} . \tag{6}$$

There exists many different representations for the photonic contribution into the structure function [9], but here we will use the form given in [5] for D^γ, $D_N^{e^+e^-}$ and $D_S^{e^+e^-\,(u)}$

$$D^\gamma(z, Q^2) = \frac{1}{2}\beta(1-z)^{\beta/2-1}\Big[1 + \frac{3}{8}\beta - \frac{\beta^2}{48}(\frac{1}{3}L + \pi^2 - \frac{47}{8})\Big] - \frac{\beta}{4}(1+z)+ \tag{7}$$

$$\frac{\beta^2}{32}\Big[-4(1+z)\ln(1-z) - \frac{1+3z^2}{1-z}\ln z - 5 - z\Big] , \quad \beta = \frac{2\alpha}{\pi}(L-1) .$$

$$D_N^{e^+e^-}(z, Q^2) = \frac{\alpha^2}{\pi^2}\Big[\frac{1}{12(1-z)}(1 - z - \frac{2m}{\varepsilon})^{\beta/2}(L_1 - \frac{5}{3})^2(1 + z^2 + \frac{\beta}{6}(L_1 - \frac{5}{3}))\Big]\theta(1 - z - \frac{2m}{\varepsilon}) , \tag{8}$$

$$D_S^{e^+e^-\,(u)} = \frac{\alpha^2}{4\pi^2}L^2\Big[\frac{2(1-z^3)}{3z} + \frac{1}{2}(1-z) + (1+z)\ln z\Big]\theta(1 - z - \frac{2m}{\varepsilon}) , \tag{9}$$

$$D_S^{e^+e^-\,(p)} = \frac{\alpha^2}{4\pi^2}L^2\Big(\frac{5(1-z)}{2} + (1+z)\ln z\Big)\theta(1 - z - \frac{2m}{\varepsilon}) , \tag{10}$$

where ε is the energy of the parent electron and $L_1 = L + 2\ln(1-z)$. The above form of the structure function $D_N^{e^+e^-}$ includes effects due to real pair production only. The correction caused by virtual pair is included in D^γ. Note that the terms containing $\alpha^2 L^3$ cancel each other in the sum $D^\gamma + D_N^{e^+e^-}$.

Instead of the photonic structure function given by Eq. (7), one can use the its iterative form [10]

$$D^\gamma(z, L) = \delta(1-z) + \sum_{k=1}^{\infty} \frac{1}{k!}\Big(\frac{\alpha L}{2\pi}\Big)^k P_1(z)^{\otimes k} , \tag{11}$$

$$\underbrace{P_1(z) \otimes \cdots \otimes P_1(z)}_{k} = P_1(z)^{\otimes k} , \quad P_1(z) \otimes P_1(z) = \int_z^1 P_1(t)P_1\Big(\frac{z}{t}\Big)\frac{dt}{t} ,$$

$$P_1(z) = \frac{1+z^2}{1-z}\theta(1-z-\Delta) + \delta(1-z)(2\ln\Delta + \frac{3}{2}) , \quad \Delta \ll 1 .$$

The iterative form (11) of D^γ does not include any effects caused by pair production. The corresponding nonsinglet part of the structure due to real and virtual pair production can be inserted into iterative form of $D^\gamma(z, L)$ by replacing $\alpha L/2\pi$ on the right side of Eq. (11) by the effective electromagnetic coupling

$$\frac{\alpha_{eff}}{2\pi} = -\frac{3}{2}\ln\Big(1 - \frac{\alpha L}{3\pi}\Big), \tag{12}$$

which is the integral of the running electromagnetic constant.

The limits of integration with respect to z_1 and z_2 in the master formula (3) can be found from the constraint on the Bjorken variable $\hat{x}$ for the partonic process

$$\hat{x} = \frac{-(\hat{k}_1 - \hat{k}_2)^2}{2p_1(\hat{k}_1 - \hat{k}_2)} = \frac{z_1 yx}{z_1 z_2 + y - 1} < 1 , \quad x = \frac{Q^2}{2p_1(k_1 - k_2)} . \tag{13}$$

By taking into account also that $z_{1,2} < 1$ and $xy = Q^2/V$, we derive from (13)

$$1 > z_2 > z_{2m} , \quad 1 > z_1 > z_{1m} , \quad z_{2m} = \frac{1-y}{z_1} + \frac{Q^2}{V} , \quad z_{1m} = \frac{V(1-y)}{V - Q^2} . \tag{14}$$

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In the framework of the leading logarithmic approximation we have to take the elastic (Born) cross–section as the hard part under the integral on the right hand side of Eq. (3)

$$\frac{d\sigma_{hard}^{\|,\perp(B)}}{d\,Q^2 d\,y} = \frac{d\sigma^{\|,\perp(B)}}{d\,Q^2}\delta(y - \frac{Q^2}{V}) \ . \tag{15}$$

In the case of the longitudinal polarization of the recoil proton, we have

$$\frac{d\sigma_{hard}^{\|(B)}}{d\,Q^2} = \frac{4\pi\alpha^2(-Q^2)}{VQ^2}(1 - \frac{Q^2}{2V})\sqrt{\frac{Q^2}{4M^2 + Q^2}}G_M^2(-Q^2) \ . \tag{16}$$

The quantity $\alpha(-Q^2)$ on the right hand side of Eq. (16) is the running electromagnetic constant that account for the effects of the vacuum polarization

$$\alpha(q^2) = \frac{\alpha}{1 - \frac{\alpha}{3\pi}\ln\frac{-q^2}{m^2}} \ .$$

For the transverse polarization of the recoil proton, the hard part of the cross–section reads

$$\frac{d\sigma_{hard}^{\perp(B)}}{d\,Q^2} = -2\frac{4\pi\alpha^2(-Q^2)}{VQ^2}\frac{M}{\sqrt{Q^2 + 4M^2}}\sqrt{1 - \frac{Q^2}{V}(1 + \tau)}G_E(-Q^2)G_M(-Q^2) \ , \quad \tau = \frac{M^2}{V} \ . \tag{17}$$

Note that in zeroth order of perturbation theory the photonic contribution into electron structure function gives an ordinary δ–function because (see also the iterative form (11))

$$\lim_{\beta\to 0}\frac{1}{2}\beta(1 - z)^{\frac{1}{2}\beta - 1} = \delta(1 - z) \ . \tag{18}$$

It is easy to see that the representation (3) reproduces the Born cross–section in this case

$$\frac{d\sigma^{\|,\perp}}{d\,Q^2 d\,y} = \int dz_1 \int dz_2 \frac{1}{z_2^2}\delta(1 - z_1)\delta(1 - z_2)\frac{d\sigma^{\|,\perp(B)}}{d\hat{Q}^2}\delta(\hat{y} - \frac{\hat{Q}^2}{\hat{V}}) = \frac{d\sigma^{\|,\perp(B)}}{d\,Q^2}\delta(y - \frac{q^2}{V}). \tag{19}$$

3 Beyond the leading approximation

We can improve the leading approximation for $d\sigma^{\|,\perp}/dQ^2dy$ given by formula (3) with $d\sigma^{\|,\perp(B)}/dQ^2dy$ as a hard part of the cross–section under the integral. It can be done by making more precise the expression namely for this hard part

$$\frac{d\sigma_{hard}^{\|,\perp}}{d\,Q^2 d\,y} = \frac{d\sigma^{\|,\perp(B)}}{d\,Q^2 d\,y} + \frac{d\sigma^{\|,\perp(1)}}{d\,Q^2 d\,y} \ . \tag{20}$$

The additional term on the right hand side of Eq. (20) takes into account RC due to real and virtual photon emission without its leading part that is absorbed by D–functions. To find $d\sigma^{\|,\perp(1)}/d\,Q^2 d\,y$, we must calculate the corresponding cross–sections of the process (1) (with virtual and soft corrections) and of the process (2), and then subtract from their sum the right hand side of formula (3) with

$$\frac{d\sigma_{hard}^{\|,\perp}}{d\,Q^2 d\,y} = \frac{d\sigma^{\|,\perp(B)}}{d\,Q^2 d\,y} \ ,$$

which appears in the same order of the perturbation theory.

We begin with the calculation of the cross–section of the radiative process (2) (the corresponding polarization calculations were performed for the case of deep inelastic scattering [11])

$$\frac{d\sigma^{\gamma(p)}}{d\,Q^2 d\,y} = \frac{2\pi\alpha^2(q^2)}{Vq^4}\frac{\alpha}{4\pi^2}L^{\gamma}_{\mu\nu}H_{\mu\nu}\frac{d^3k}{k_0}\frac{d^3p_2}{p_{20}}\delta(p_1 + k_1 - k_2 - p_2 - k) \ , \tag{21}$$

where $q = k_1 - k_2 - k = p_2 - p_1$. In further we will be interested in the polarization dependent parts of the leptonic $L_{\mu\nu}$ and hadronic $H_{\mu\nu}$ tensors and assume that the degree of initial electron polarization is equal to 1. In this case we have

$$H_{\mu\nu} = -iM\epsilon_{\mu\nu\lambda\rho}q_\lambda[-G_E(q^2)A_\rho + \frac{2(G_E(q^2) - G_M(q^2))}{4M^2 - q^2}(Ap_1)p_{1\rho}]G_M(q^2) \ , \tag{22}$$

$$L^{\gamma}_{\mu\nu} = -2i\epsilon_{\mu\nu\lambda\rho}q_\lambda[k_{1\rho}R_t + k_{2\rho}R_s] \ , \tag{23}$$

$$R_t = \frac{u+t}{st} - 2m^2(\frac{1}{s^2} + \frac{1}{t^2}) \ , \ \ R_s = \frac{u+s}{st} - 2m^2\frac{s_t}{ut^2} \ , s_t = \frac{-u(u+Vy)}{u+V} \ ,$$

where A is the 4–vector of the recoil proton polarization and we use the following notation for invariants

$$u = (k_1 - k_2)^2 \ , \ \ s = 2kk_2 \ , \ \ t = -2kk_1 \ , \ \ q^2 = u + s + t \ , \ \ Q^2 = -u \ .$$

It is convenient to express the recoil proton polarization 4–vector A in terms of the particle 4–momenta and Lorentz invariants. Below we use the following parameterization for $A^{\parallel}$ and $A^{\perp}$

$$A^{\parallel}_\mu = \frac{2M^2q_\mu - q^2p_{2\mu}}{MQ_{\parallel}} \ , \ \ Q_{\parallel} = \sqrt{-q^2(4M^2 - q^2)} \ , \tag{24}$$

$$A^{\perp}_\mu = \frac{2[2M^2k_1q + q^2k_1p_1]p_{2\mu} - 2[2M^2k_1q - q^2k_1p_2]p_{1\mu} + q^2(q^2 - 4M^2)k_{1\mu}}{2Q_{\perp}} \ , \tag{25}$$

$$Q_{\perp} = \sqrt{q^2[q^2M^2(k_1p_1 + k_1p_2)^2 + (2M^2k_1q - q^2k_1p_2)(2M^2k_1q + q^2k_1p_1)]} \ .$$

$$2k_1p_2 = V + u + t \ , \ \ 2k_1q = u + t \ .$$

It is easy to verify that 4–vector $A^{\parallel}$ in the rest frame of the recoil proton has components $(0, \vec{n})$, where 3–vector $\vec{n}$ has orientation of the recoil proton 3–momentum in laboratory system. One can verify also that $A^{\perp}A^{\parallel} = 0$ and in the rest frame of the recoil proton

$$A^{\perp} = (0, \vec{n}_{\perp}) \ , \ \ \vec{n}^2_{\perp} = 1 \ , \ \ \vec{n}\vec{n}_{\perp} = 0 \ ,$$

where the 3–vector $\vec{n}_{\perp}$ is within the plane $(\vec{k}_1, \vec{p}_2)$ in the laboratory system.

For the case of longitudinal polarization, the contraction of leptonic and hadronic tensors yields

$$\frac{L^{\gamma}_{\mu\nu}H_{\mu\nu}}{q^4} = -\frac{2m^2}{s^2}(q_s^2 + 2V)F(q_s^2) - \frac{2m^2}{t^2}(u + 2V)(1 + \frac{s_tq_t^2}{u^2})F(q_t^2) + \tag{26}$$

$$\{[\frac{1}{tu}[(u^2 + q^4)(u + 2V) - 2q^2(q^2 - q_t^2)(u + V)] + \frac{1}{sq_s^2}[(q^4 + u^2)(q_s^2 + 2V) - 2q^2V(q^2 - q_s^2)]\}\frac{F(q^2)}{q^2 - u} \ ,$$

where

$$q_t^2 = u + s_t = \frac{uV(1 - y)}{u + V} \ , \ \ q_s^2 = u + t_s = \frac{uV}{V(1 - y) - u} \ , \ \ t_s = \frac{u(u + Vy)}{V(1 - y) - u} \ ,$$

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$$F(q^2) = -G_M^2(q^2)\frac{1}{q^2}\sqrt{\frac{-q^2}{4M^2 - q^2}} \ .$$

The physical meaning of quantities q_t^2 and q_s^2 is as follows: q_t^2 and q_s^2 are the values of q^2 in the cases of the initial–state and final–state collinear radiation, respectively. When writing the formula (26), we took into account the fact that terms containing the electron mass squared contribute only in collinear kinematics.

To separate the contribution into the right-hand side of Eq. (26) due to collinear radiation for the pole–like terms, we apply the operations $\hat{P}_t$ and $\hat{P}_s$,

$$\frac{1}{t}f(q^2, u, t, s) = \frac{1}{t}(1 - \hat{P}_t + \hat{P}_t)f(q^2, u, t, s) \ , \quad \hat{P}_t f(q^2, u, s, t) = f(q_t^2, u, s_t, 0)$$

for arbitrary nonsingular function at $t \to 0$ and similarly for $1/s$ terms. Therefore, we can rewrite the right hand side of Eq. (26) in the form

$$\left\{ -\frac{2m^2}{s^2}(q_s^2 + 2V)\hat{P}_s - \frac{2m^2}{t^2}(u + 2V)(1 + \frac{s_t q_t^2}{u})\hat{P}_t \right\}F(q^2) + \left\{ \frac{(u + 2V)(u^2 + q_t^4)}{ut}\hat{P}_t + \quad (27) \right.$$

$$\frac{(q_s^2 + 2V)(u^2 + q_s^4)}{q_s^2 s}\hat{P}_s + \frac{1 - \hat{P}_t}{ut}[(u + 2V)(u^2 + q^4) - 2q^2(q^2 - q_t^2)(u + V)] +$$

$$\left. \frac{1 - \hat{P}_s}{q_s^2 s}[(q_s^2 + 2V)(u^2 + q^4) - 2q^2 V(q^2 - q_s^2)] \right\} \frac{F(q^2)}{q^2 - u} \ .$$

For the case of transverse polarization the contraction of leptonic and hadronic tensors is more complicated,

$$\frac{1}{q^4}L_{\mu\nu}^\gamma H_{\mu\nu} = \{[q^2(u + t + 2V)^2 + (4M^2 - q^2)(u + t)^2]R_t + [q^2(u + t + 2V)(t - q^2 + 2V(1 - y)) + \quad (28)$$

$$(4M^2 - q^2)(uq^2 - st)]R_s\}\frac{G_E(q^2)G_M(q^2)}{q^4}\sqrt{\frac{-q^2 M^2}{(4M^2 - q^2)(-q^2 V(V + u + t) - M^2(u + t)^2)}} \ .$$

The expression in the round brackets on the right hand side of Eq. (28) can be rewritten in the form suitable for the photon angular integration as follows:

$$-2[q^2 V y + 4M^2(q^2 + u)] - \frac{2m^2}{s^2}4V^2 q_s^2 K_s\hat{P}_s - \frac{2m^2}{t^2}4V^2 q_t^2(1 + \frac{s_t q_t^2}{u^2})K_t\hat{P}_t + \quad (29)$$

$$\frac{1}{t}\left[\frac{4V^2 q_t^2(u^2 + q_t^4)}{u(q_t^2 - u)}K_t\hat{P}_t + (1 - \hat{P}_t)\frac{q^2}{u(q^2 - u)}[4V^2(u^2 + q^4)K_q - 2q^2(q^2 - q_t^2)(u + 2V)(u + V)]\right] +$$

$$\frac{1}{s}\left[\frac{4V^2(u^2 + q_s^4)}{(q_s^2 - u)}K_s\hat{P}_s + (1 - \hat{P}_s)\frac{q^2 V}{q_s^2(q^2 - u)}[4V(u^2 + q^4)K_s - 2V(q^2 - q_s^2)(2Vq^2 - u^2)]\right] \ ,$$

$$K_s = 1 + \frac{q_s^2}{V}(1 + \tau) \ , \quad K_t = 1 + \frac{u}{V} + \frac{u^2 \tau}{Vq_t^2} \ , \quad K_q = 1 + \frac{u}{V} + \frac{u^2 \tau}{Vq^2} \ .$$

To perform the photon angular integration we choose the system $\vec{k}_1 + \vec{p}_1 - \vec{k}_2 = 0$. In this system the energies of particles are

$$k_0 = \frac{a}{2\sqrt{R}} \ , \quad k_{10} = \frac{u + V}{2\sqrt{R}} \ , \quad k_{20} = \frac{V(1 - y) - u}{2\sqrt{R}} \ , \quad p_{10} = \frac{2M^2 + Vy}{2\sqrt{R}} \ , \quad p_{20} = \frac{R + M^2}{2\sqrt{R}} \ , \quad (30)$$

$$a = u + Vy \ , \quad R = a + M^2 \ .$$

Taking the OZ axis along the initial proton 3–momentum in the chosen system we also have

$$c_k = \cos\theta_k = \frac{2M^2 - 2p_{10}p_{20} - q^2}{2|\vec{p}_1||\vec{p}_2|} \ , \quad c_2 = \cos\theta_2 = \frac{2k_{20}p_{10} - V(1-y)}{2|\vec{p}_1||\vec{k}_2|} \ , \tag{31}$$

$$c_1 = \cos\theta_1 = \frac{2k_{10}p_{10} - V}{2|\vec{p}_1||\vec{k}_1|} \ , \ |\vec{p}_1| = \frac{\sqrt{V^2y^2 - 4uM^2}}{2\sqrt{R}} \ , \ |\vec{p}_2| = k_0 \ ,$$

where $\theta_1(\theta_2)$ is the polar angle of the initial (scattered) electron and θ_k is the photon polar angle. Besides Eqs. (30) and (31) we will use the relation

$$\frac{d^3k}{k_0}\frac{d^3p_2}{p_{20}}\delta(k_1 + p_1 - k_2 - k - p_2) = \frac{a}{2R}d\varphi \, d\cos\theta_k \ . \tag{32}$$

Let us concentrate on the case with longitudinal polarization of the recoil proton. For the terms containing m^2/s^2, m^2/t^2, $\hat{P}_t/t$ and $\hat{P}_s/s$ we can use the following formulae

$$\int\frac{m^2d\varphi d\cos\theta_k}{2\pi s^2} = \int\frac{m^2d\varphi d\cos\theta_k}{2\pi t^2} = \frac{2R}{a^2} \ , \quad \int\frac{d\varphi d\cos\theta_k}{2\pi s} = \frac{2R}{a(V(1-y)-u)}(L_s + L) \ , \tag{33}$$

$$\int\frac{d\varphi d\cos\theta_k}{2\pi(-t)} = \frac{2R}{a(u+V)}(L_t + L) \ , \ L_s = \ln\frac{(V(1-y)-u)^2}{-uR} \ , \ L_t = \ln\frac{(V+u)^2}{-uR} \ .$$

Terms which contain $(1 - \hat{P}_t)$, $(1 - \hat{P}_s)$ operators can be integrated over the azimuthal angle and keep the integration with respect to q^2 using $d\cos\theta_k = d\,q^2/2|\vec{p}_1||\vec{p}_2|$,

$$\int\frac{d\,\varphi}{2\pi s 2|\vec{p}_1||\vec{p}_2|} = \frac{2R}{a|q^2 - q_s^2|(V(1-y)-u)} \ , \quad \int\frac{d\,\varphi}{2\pi(-t)2|\vec{p}_1||\vec{p}_2|} = \frac{2R}{a|q^2 - q_t^2|(V+u)} \ . \tag{34}$$

The limits of q^2–integration in this case can be derived from the restriction on $\cos\theta_k$ in the chosen system; $|\cos\theta_k| < 1$. This restriction leads to the relation

$$q_-^2 < q^2 < q_+^2 \ , \quad q_\pm^2 = \frac{1}{2R}[2uM^2 - Vy(u+Vy) \pm (u+Vy)\sqrt{V^2y^2 - 4uM^2}] \ . \tag{35}$$

By using Eqs. (33), (34) and (35) we can write the cross–section of the radiative process (2) in the case of longitudinal polarization of the recoil proton as follows

$$\frac{d\sigma^{\|\gamma}}{d\,Q^2 d\,y} = \frac{2\alpha}{V}\Big\{-\frac{q_s^2 + 2V}{u+Vy}\hat{P}_s - \frac{(u+2V)(u^2 + s_t q_t^2)}{u^2(u+Vy)}\hat{P}_t - \tag{36}$$

$$-[1 + L_t + (L-1)]\frac{(u+2V)(u^2+q_t^4)}{2u(u+V)(q_t^2-u)}\hat{P}_t + [1 + L_s + (L-1)]\frac{(q_s^2+2V)(u^2+q_s^4)}{2q_s^2(V(1-y)-u)(q_s^2-u)}\hat{P}_s +$$

$$\int_{q_-^2}^{q_+^2}\Big[-\frac{d\,q^2}{|q^2 - q_t^2|}(1 - \hat{P}_t)\frac{(u+2V)(u^2+q^4) - 2q^2(q^2-q_t^2)(u+V)}{2u(u+V)(q^2-u)} +$$

$$\frac{d\,q^2}{|q^2 - q_s^2|}(1 - \hat{P}_s)\frac{(q_s^2+2V)(u^2+q^4) - 2q^2V(q^2-q_s^2)}{2q_s^2(V(1-y)-u)(q^2-u)}\Big]\Big\}\alpha^2(q^2)F(q^2)\theta(y + \frac{u}{V} - \frac{2M\Delta\varepsilon}{V}) \ .$$

The appearance of the θ–function on the right side of Eq. (36) is connected with the restriction on the photon hardness in the radiative process (2)

$$k_0 = \frac{u + Vy}{2\sqrt{M^2 + u + Vy}} > \Delta\varepsilon \quad \rightarrow \quad y > -\frac{u}{V} + \frac{2M\Delta\varepsilon}{V} \ , \tag{37}$$

where $\Delta\varepsilon$ is the the minimal photon energy in the chosen coordinate system.

To be complete, we should also take into account the RC due to virtual and soft (with the energy smaller than $\Delta\varepsilon$) photon emission to the cross–section of the elastic process (1). It can be written as (see, for example, [6])

$$\frac{d\sigma^{\|(V+S)}}{dQ^2dy} = \frac{4\pi\alpha^2(-Q^2)}{V}(1 - \frac{Q^2}{2V})F(-Q^2)\frac{\alpha}{2\pi}\Big[2(L-1)(\ln\frac{4M^2(\Delta\varepsilon)^2}{V(u+V)} + \frac{3}{2}) - \tag{38}$$

$$-1 - \frac{\pi^2}{3} - \ln^2\frac{u+V}{V} - 2f(\frac{u+V+u\tau}{u+V})\Big]\delta(y - \frac{Q^2}{V}) \ , \quad f(x) = \int\limits_0^x\frac{dx}{x}\ln(1-x).$$

Therefore, the sum of the cross–sections of the processes (1) and (2) is defined by the formula

$$\frac{d\sigma^{\|(B)}}{dQ^2dy} + \frac{d\sigma^{\|\gamma}}{dQ^2dy} + \frac{d\sigma^{\|(S+V)}}{dQ^2dy} \ . \tag{39}$$

To include the hard cross–section into the electron structure function representation (3) in the form (39) and get rid of the double counting, we must remove from the sum (39) the contribution which arises in the representation (3) in the first order with respect to fine structure constant α at

$$\frac{d\sigma^{\|}_{hard}}{dQ^2dy} = \frac{d\sigma^{\|(B)}}{dQ^2dy} \ .$$

The procedure for finding this contribution is described in [6]. We can verify that it equals to

$$\frac{2\alpha}{V}\Big\{(L-1)\Big[-\frac{(u+2V)(u^2+q_t^4)}{2u(u+V)(q_t^2-u)}\hat{P}_t + \frac{(q_s^2+2V)(u^2+q_s^4)}{2q_s^2(V(1-y)-u)(q_s^2-u)}\hat{P}_s\Big]\alpha^2(q^2)F(q^2) \tag{40}$$

$$\times\theta(y + \frac{u}{V} - \frac{2M\Delta\varepsilon}{V}) + 2(L-1)(\ln\frac{4M^2(\Delta\varepsilon)^2}{V(u+V)} + \frac{3}{2})(1 - \frac{Q^2}{2V})\alpha^2(-Q^2)F(-Q^2)\delta(y + \frac{u}{V})\Big\} \ .$$

Thus, we can write the final result for the $d\sigma^{\|}_{hard}/dQ^2dy$ in the following very compact form

$$\frac{d\sigma^{\|}_{hard}}{dQ^2dy} = \frac{d\sigma^{\|(B)}}{dQ^2dy}\Big\{1 + \frac{\alpha}{2\pi}\Big[-1 - \frac{\pi^2}{3} - \ln^2\frac{u+V}{V} - 2f(\frac{u+V+u\tau}{u+V})\Big]\Big\}+ \tag{41}$$

$$\frac{2\alpha}{V}\Big\{\frac{(u+2V)(q_t^2-u)}{2u(u+V)}\hat{P}_t + \frac{(q_s^2+2V)(q_s^2-u)}{2uV}\hat{P}_s + P\int\limits_{q_-^2}^{q_+^2}\frac{dq^2}{q^2-u}\Big[\frac{1}{|q^2-q_s^2|}(1 - \hat{P}_s)$$

$$\times\frac{(q_s^2+2V)(u^2+q^4) - 2q^2V(q^2-q_s^2)}{2q_s^2(V(1-y)-u)} -$$

$$\frac{1}{|q^2-q_t^2|}(1 - \hat{P}_t)\frac{(u+2V)(u^2+q^4) - 2q^2(u+V)(q^2-q_t^2)}{2u(V+u)}\Big]\Big\}\alpha^2(q^2)F(q^2)\theta(y + \frac{u}{V}) \ ,$$

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where P is the symbol of the principal value integration. When writing the last formula, we used the following relations

$$P\int_{q_-^2}^{q_+^2}\frac{d\,q^2[f(q^2)-f(q_t^2)]}{|q^2-q_t^2|(q^2-u)}=\frac{f(q_t^2)}{q_t^2-u}L_t+\int_{q_-^2}^{q_+^2}\frac{d\,q^2}{|q^2-q_t^2|}\Big(\frac{f(q^2)}{q^2-u}-\frac{f(q_t^2)}{q_t^2-u}\Big)\,,\qquad(42)$$

$$P\int_{q_-^2}^{q_+^2}\frac{d\,q^2[f(q^2)-f(q_s^2)]}{|q^2-q_s^2|(q^2-u)}=\frac{f(q_s^2)}{q_s^2-u}L_s+\int_{q_-^2}^{q_+^2}\frac{d\,q^2}{|q^2-q_s^2|}\Big(\frac{f(q^2)}{q^2-u}-\frac{f(q_s^2)}{q_s^2-u}\Big)\,,\qquad(43)$$

where the symbol P indicates how one shall integrate the unphysical singularity at $q^2=u$. These relations allow to see that infrared singularities of separate terms in $d\sigma^{\|(1)}/dQ^2dy$ exactly cancel each other. That is why we omitted from argument of the θ–function on the right side of Eq. (41) the term $-2M\Delta\varepsilon/V$. For numerical calculations the symbol P can be understood as

$$P\int_{q_-^2}^{q_+^2}\frac{d\,q^2}{q^2-u}F(q^2)=\int_{q_-^2}^{q_+^2}\frac{d\,q^2}{q^2-u}(F(q^2)-F(u))+F(u)\log\frac{q_+^2-u}{q_-^2-u}$$

The hard part of the cross–section in the case of transverse polarization of the recoil proton can be derived in full analogy with the above. The main difference is caused by the fact that the vector of transverse polarization has complicated dependence on the photon azimuthal angle ϕ and therefore even ϕ integration becomes nontrivial. The straightforward calculations give

$$\frac{d\sigma_{hard}^{\perp}}{d\,Q^2dy}=\frac{d\sigma^{\perp(B)}}{d\,Q^2dy}\{1+\frac{\alpha}{2\pi}[-1-\frac{\pi^2}{3}-\ln^2\frac{u+V}{V}-2f(\frac{u+V+u\tau}{u+V})]\}+\qquad(44)$$

$$\frac{2\alpha}{V}\{\Big[\frac{2(q_t^2-u)V}{u(u+V)}+\frac{2V(u^2+q_t^4)}{u^2(u+Vy)}L_t\Big]\frac{K_t\hat{P}_t}{q_t^2}+\Big[\frac{2(q_s^2-u)}{u}+\frac{2V(u^2+q_s^4)}{uq_s^2(u+Vy)}L_s\Big]\frac{K_s\hat{P}_s}{q_s^2}+$$

$$\int_{q_-^2}^{q_+^2}\frac{d\,q^2}{\sqrt{V^2y^2-4uM^2}}\int_0^{2\pi}\frac{d\varphi}{2\pi}\Big[\frac{[-yq^2-4\tau(u+q^2)]}{q^4}+\frac{1-\hat{P}_t}{t}\Big(\frac{2V(u^2+q^4)}{uq^2(q^2-u)}K_q-$$

$$\frac{(u+2V)(u+V)(q^2-q_t^2)}{uV(q^2-u)}\Big)+\frac{1-\hat{P}_s}{s}\Big(\frac{2V(u^2+q^4)}{q_s^2q^2(q^2-u)}K_s+\frac{(u^2-2q^2V)(q^2-q_s^2)}{q_s^2q^2(q^2-u)}\Big)\Big]\}$$

$$\times\alpha^2(q^2)\sqrt{\frac{M^2}{4M^2-q^2}}(1+\frac{u+t}{V}+\frac{(u+t)^2\tau}{Vq^2})^{-1/2}G_E(q^2)G_M(q^2)\theta(y+\frac{u}{V})\,.$$

For invariants s and t on the right side of Eq. (44), we can neglect the electron mass and use here the simplified expressions

$$s=c_{2i}-s_i\cos\varphi,\quad -t=c_{1i}-s_i\cos\varphi,\qquad(45)$$

$$c_{1i}=2k_0k_{10}[1-\cos\theta_1\cos\theta_k],\quad c_{2i}=2k_0k_{20}[1-\cos\theta_2\cos\theta_k],$$

$$s_i=2k_0k_{10}\sin\theta_1\sin\theta_k=2k_0k_{20}\sin\theta_2\sin\theta_k$$

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The integrals over ϕ can be performed in terms of elliptic functions $\mathcal{K}$ and Π

$$\int_0^{2\pi} \frac{d\varphi}{2\pi}\left(1 + \frac{u+t}{V} + \frac{(u+t)^2\tau}{Vq^2}\right)^{-1/2} = J_0 = \frac{2}{\pi\sqrt{X}}\mathcal{K}(\kappa) \tag{46}$$

$$\int_0^{2\pi} \frac{d\varphi}{2\pi t}\left(1 + \frac{u+t}{V} + \frac{(u+t)^2\tau}{Vq^2}\right)^{-1/2} = \frac{J_t}{|q^2 - q_t^2|} =$$

$$= -\frac{B_t(1+b_t)\sqrt{\lambda\bar{y}}}{2(V+u)|q^2 - q_t^2|\sqrt{X}}\left(\frac{2}{\pi}\sqrt{1 - b_{1t}}\mathcal{K}(\kappa) + \frac{B_t}{b_{1t}\bar{y}}\frac{1 - \Lambda(\epsilon,\kappa)}{\sqrt{1 - \kappa^2/b_{1t}}}\right),$$

$$\int_0^{2\pi} \frac{d\varphi}{2\pi s}\left(1 + \frac{u+t}{V} + \frac{(u+t)^2\tau}{Vq^2}\right)^{-1/2} = \frac{J_s}{|q^2 - q_s^2|} =$$

$$= \frac{B_s(1+b_s)\sqrt{\lambda\bar{y}}}{2(V-a)|q^2 - q_s^2|\sqrt{X}}\left(\frac{2}{\pi}\sqrt{1 - b_{1s}}\mathcal{K}(\kappa) + \frac{B_s}{b_{1s}\bar{y}}\frac{1 - \Lambda(\epsilon,\kappa)}{\sqrt{1 - \kappa^2/b_{1s}}}\right),$$

where

$$X = (1+x_+)(1-x_-)\frac{M^2 s_i^2}{-q^2 V^2}, \quad \kappa^2 = \frac{2(x_+ - x_-)}{(1+x_+)(1-x_-)}, \quad \bar{y} = \frac{-x_- + 1}{-x_- - 1},$$

$$\begin{aligned}
2M^2 s_i x_\pm &= 2M^2(-q^2 + c_{2i}) - Vq^2(1 \pm \sqrt{1 - 4M^2/q^2}) \\
&= 2M^2(-u + c_{1i}) - Vq^2(1 \pm \sqrt{1 - 4M^2/q^2})
\end{aligned} \tag{47}$$

$$b_{1s,t} = \frac{1 + \bar{y} - b_{s,t} + b_{s,t}\bar{y}}{\bar{y}(1+b_{s,t})}, \quad b_{s,t} = \frac{c_{2,1i}}{s_i}, \quad B_{s,t} = b_{1s,t}\bar{y} + 1 - \bar{y}.$$

The function $\Lambda(\epsilon,\kappa)$ ($\epsilon = \arcsin((1-b_1)/(1-\kappa^2))$) is non-singular Heuman's Lambda function varying from 0 to 1 (see [12] for details and exact definitions). It is related with complete elliptic integral $\Pi(b_1,\kappa)$ of the third kind

$$\frac{2}{\pi}\Pi(b_1,\kappa) = \frac{1 - \Lambda(\epsilon,\kappa)}{\sqrt{1-b_1}\sqrt{1-\kappa^2/b_1}} + \frac{2}{\pi}\mathcal{K}(\kappa) \tag{48}$$

For $\epsilon \to 0$ (or $b_1 \to 1$) this function goes to zero. In the last formula singular behavior of $\Pi(b_1,\kappa)$ for $b_1 \to 1$ is extracted explicitly in the first term. This limit corresponds to collinear radiation:

$$\begin{aligned}
\sqrt{1 - b_{1t}} &= \frac{u+V}{(1+b_t)s_i\sqrt{\bar{y}\lambda}}|q^2 - q_t^2| \\
\sqrt{1 - b_{1s}} &= \frac{V-a}{(1+b_s)s_i\sqrt{\bar{y}\lambda}}|q^2 - q_s^2|
\end{aligned} \tag{49}$$

where $\lambda = y^2 V^2 - 4M^2 u$. As a result of substituting Eqs (46–48) into the formula for hard cross–section (44) we arrive at the same structure of singularities as in the longitudinal case (36). In the collinear limit $q^2 \to q_{t,s}^2$, we have

$$\bar{y}X(1 - \kappa^2/b_{1t,s}) \to K_{t,s} \qquad b_{t,s}, b_{1t,s}, B_{t,s} \to 1, \qquad \Lambda(\epsilon,\kappa) \to 0$$

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These limiting formulae allows us to use relations (42,43) to write the final expression for hard cross section in such a form that provides an explicit cancellation of infrared divergence in the same way as in the case of the longitudinal polarization.

Combining all results together, we obtain the final formula for cross section in the transversely polarized case:

$$\frac{d\sigma_{hard}^{\perp}}{d\,Q^2 d\,y} = \frac{d\sigma^{\perp(B)}}{d\,Q^2 d\,y}\{1 + \frac{\alpha}{2\pi}[-1 - \frac{\pi^2}{3} - \ln^2\frac{u+V}{V} - 2f(\frac{u+V+u\tau}{u+V})]\}+ \tag{50}$$

$$\frac{2\alpha}{V}\{\frac{2(q_t^2-u)V}{u(u+V)}\frac{K_t\hat{P}_t}{q_t^2} + \frac{2(q_s^2-u)}{u}\frac{K_s\hat{P}_s}{q_s^2} + P\int_{q_-^2}^{q_+^2}\frac{d\,q^2}{\sqrt{\lambda}(q^2-u)}\Big[\frac{[-yq^2-4\tau(u+q^2)]}{q^4}(q^2-u)J_0$$

$$+\frac{1-\hat{P}_t}{|q^2-q_t^2|}J_t\Big(\frac{2V(u^2+q^4)}{uq^2}K_q - \frac{(u+2V)(u+V)(q^2-q_t^2)}{uV}\Big)+$$

$$+\frac{1-\hat{P}_s}{|q^2-q_s^2|}J_s\Big(\frac{2V(u^2+q^4)}{q_s^2q^2}K_s + \frac{(u^2-2q^2V)(q^2-q_s^2)}{q_s^2q^2}\Big)\Big]\}$$

$$\times\alpha^2(q^2)\sqrt{\frac{M^2}{4M^2-q^2}}G_E(q^2)G_M(q^2)\theta(y+\frac{u}{V})\ .$$

The theoretical formula for the ratio of longitudinal and transverse polarizations of the recoil proton that was measured in recent experiments [2, 3] is defined by the ratio of the right–hand side of Eq. (3) for longitudinal polarization (with (41) as the hard cross-section under integral sign) and for transverse one (with (44) as the hard cross–section). This high precision formula takes into account model independent RC with all the leading and the main part of the next–to–leading corrections, and has accuracy at the level of per mile.

4   Numerical analysis

The ratio of proton elastic formfactors G_{ep}/G_{mp} measured experimentally [2, 3] is related to the ratio of recoiled proton polarization components. At the Born level (i.e. without RC) the ratio of polarizations is defined by the ratio of spin dependent cross section given by (16) and (17):

$$\frac{P_T}{P_L} = \frac{\sigma_T^0}{\sigma_L^0} \tag{51}$$

The photon spectrum can be defined as a function of missing mass $W_m^2 = yV - Q^2$ (either y or photon energy in the chosen frame E_γ) of observed cross section $\sigma_{T,L}(W_M^2)$ defined by master equation (3). An integral over y gives a radiative correction to recoil polarizations and to their ratio. Let us define the following quantities

$$R_{T,L}(W_m^2) = \frac{\sigma_{T,L}(W_m^2)}{\sigma_{T,L}^0}, \quad r(W_m^2) = \frac{R_T(W_m^2)}{R_L(W_m^2)}, \quad r_{T,L} = \int\frac{dW_m^2}{V}R_{T,L}(W_m^2), \quad r = \frac{r_T}{r_L}. \tag{52}$$

In Figure 1 the $R_{T,L}$ as a function of missing mass is presented. For very small values of missing mass or alternatively for $y \to Q^2/V$ the cross sections reproduce the δ-function behavior. In the limit (18) there are three delta-functions (from D^u, D^p and from y-dependence of Born cross section) and only two integration. So we have behavior as in Eq. (18) in this limit. Only the factorization part is important here, so both longitudinal and transverse $R's$ are
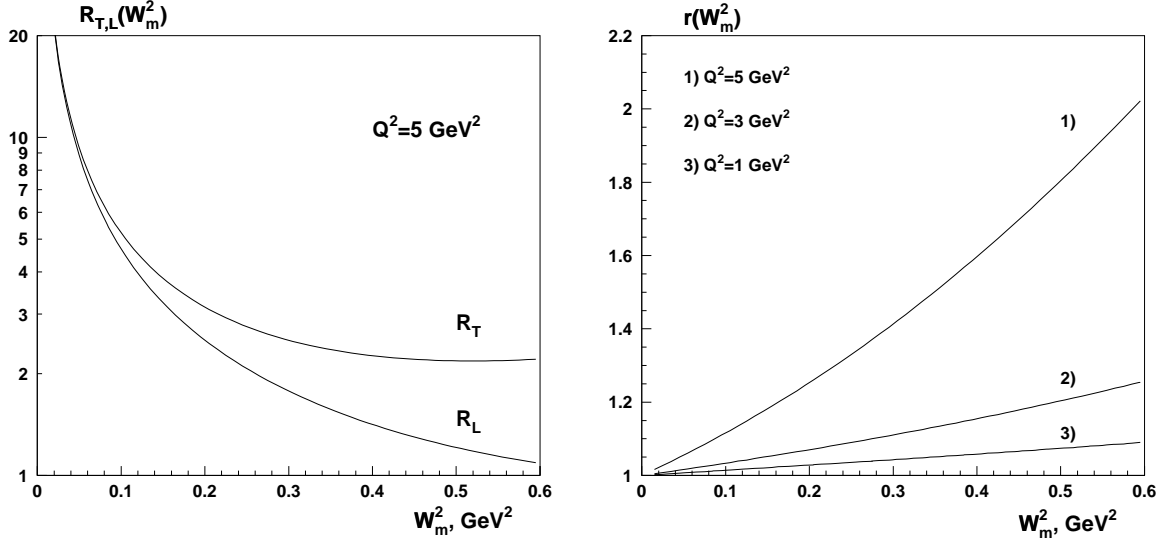
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Figure 1: Longitudinal and transverse polarization parts of cross sections normalized to born ones (left plot) and their ratios (right plot) (see Eq.(52) for exact definitions) as a function of missing mass squared for beam energy 4.26 GeV ($V=8$GeV^2).

practically the same. For larger values of W_m^2 (or y) nonfactorized part contribution becomes important. It can be seen from Figure 1b, where ratios of these spectrum are presented.

Figure 2 presents the results integrated over $dy = dW_m^2/V$. This integration has to be performed up to some specific values of a cut on the missing mass which is defined by experimental conditions. Using the hard cut leads to negative values RC (or $r_{T,L}$ becomes less then one), because the contribution of loops, which is usually negative, dominates in this case. If the positive contribution of hard photon radiation is allowed by using less stringent cuts, the radiative correction to polarized parts of cross section goes up and can exceed several tens of per cents. The right plot in Figure 2 gives a radiative correction factor to the polarization ratio or the measured ratio of formfactors. One can see that the radiative correction to it is rising not only with the increasing value of the cut but also with increasing Q^2. Within the kinematical conditions of JLAB, the radiative correction is at the level of several per cents or smaller if the hard cut on missing mass (or missing energy) is used.

5 Discussion and Conclusion

In this paper we calculated radiative corrections to observable quantities in elastic electron-proton scattering where polarization of the final proton is measured. Observable cross section of this process has to include QED loop effects and contributions of radiation of real photons and electron-positron pair creation from leptonic line. In this paper the method of structure functions is applied for this calculation. Within this approach it is possible to calculate the contributions of leading and next-to-leading order in all order of perturbation theory. Obtained explicit formulae are free from infrared divergence and can be used for straightforward numerical analysis. This numerical analysis was done for the kinematical condition of current and future
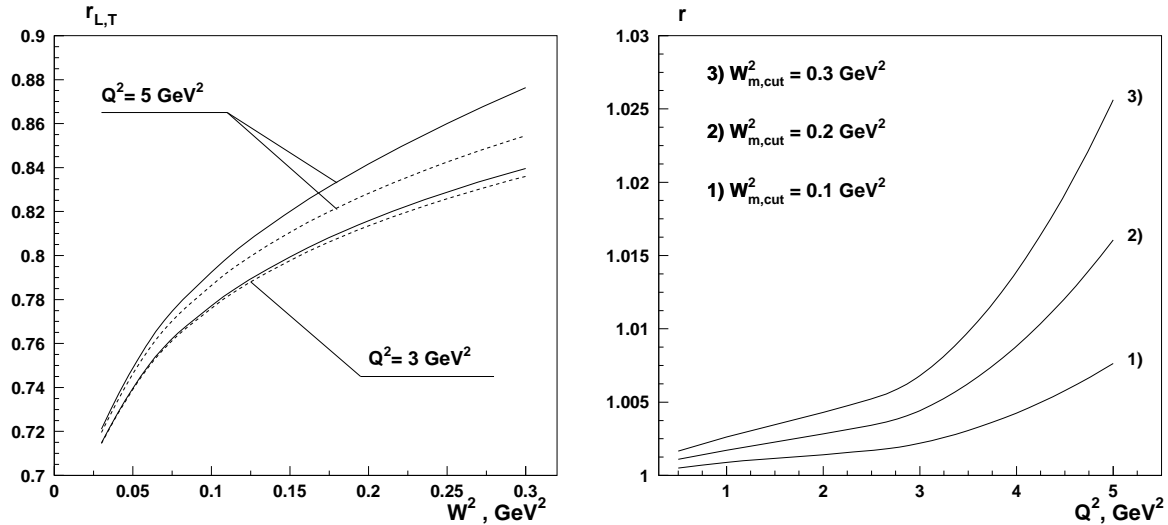
Figure 2: Radiative correction to recoil polarization rations $r_{T,L}$ (left plot) and r (right plot) (52) within the kinematical conditions of JLAB, as a function of Q^2 and value of a cut on missing mass for beam energy 4.26 GeV (V=8GeV^2). Solid (dashed) line on the left plot shows r_T (r_L).

experiments at JLAB. The concrete values of radiative correction factors were calculated. It was shown that radiative correction to observable ratio is at the per cent level.

We note that the problem was solved for the case when kinematical variable Q^2 is reconstructed via electron momentum measured. Another way is possible for which this variable is calculated using the measurement of final proton momentum. This case requires another treatment, which will be done elsewhere. Also the present calculation does not include effects due to two-photon coupling to the proton.

The target considered in this paper is proton, however the results can be straightforwardly generalized to the case when a nuclear target is used instead. In this case the effects of Fermi motion and finite momentum of spectator nucleon system have to be taken into account.

Acknowledgements

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